

A. Review

A polygon is *convex* 凸 if each of its interior angles is less than a straight angle (i.e. 180°). For example, the hexagon (6) in Fig. 13 is convex. On the other hand, when one or more of the interior angles of a polygon is reflex, i.e. greater than 180° , the polygon is *concave* 凹. For example, the pentagon (5) in Fig. 14 is concave. We are going to discuss only convex polygons at this level.

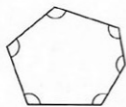


Fig. 13 Convex hexagon (6)



Fig. 14 Concave pentagon (5)

A polygon is *equilateral* 等边 if all its sides are of equal length, and is *equiangular* 等角 if all its angles are equal. A *regular polygon* 正多边形 is both equilateral and equiangular.

Note: An equilateral triangle must also be equiangular; but this is NOT true for other polygons.

e.g. Fig. 15 shows an equilateral hexagon (6). The marked angles a and b are obviously not equal, hence the equilateral hexagon is not equiangular.

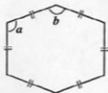


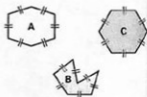
Fig. 15

In this chapter, the number given in brackets after the name of a polygon indicates the number of sides of the polygon.

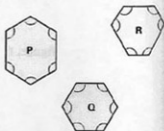


Try This

1. These hexagons have 6 equal sides. Which one is a regular hexagon?



2. These hexagons have 6 equal angles. Which one is a regular hexagon?



B. Sum of Interior Angles of A Polygon

Fig. 16 shows a quadrilateral ABCD.

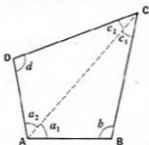


Fig. 16

If we draw a diagonal, say AC, then the diagonal divides the quadrilateral into 2 triangles, namely $\triangle ABC$ and $\triangle ADC$. With the notation given in the figure, we have

$$\begin{aligned} \text{the sum of all the interior angles} &= (a_1 + a_2) + d + (c_1 + c_2) + b \\ &= (a_1 + b + c_1) + (a_2 + d + c_2) \\ &= 180^\circ + 180^\circ \\ &= 360^\circ \end{aligned}$$

In fact, any quadrilateral can be divided into two triangles by a diagonal. Hence, the sum of the interior angles of a quadrilateral is the sum of the interior angles of 2 triangles, i.e. $360^\circ (= 2 \times 180^\circ)$.

Recall:

A line joining two nonadjacent vertices of a polygon is called a diagonal of the polygon.

- ◀ (i) $a_1 + b + c_1 = 180^\circ$
(\angle sum of $\triangle ABC$)
(ii) $a_2 + d + c_2 = 180^\circ$
(\angle sum of $\triangle ADC$)