Chapter Overview

The focus of this chapter is on Pythagoras' theorem and its applications. But students should be proficient in working with square roots before they proceed to learn the theorem.

Pythagoras' theorem is a very important theorem, both for its own sake and for its many applications in other fields within and outside Mathematics.

Students are expected to know the Pythagoras' theorem and be able to apply it in simple situations. They should be able to follow a proof of the theorem, but they are not expected to be able to produce a proof of the theorem at this stage. They will also need to know the converse of the theorem, though they are not expected to solve many problems involving the converse of the theorem.

Calculators are needed in most of the sections in this chapter, therefore students should bring their calculators to class for this chapter.

CHAPTER 5

5.1 Square Roots

TG In Fig. 1, the length of each side of the square is 4 units.

Thus

area of the square = (4×4) sq. units = 16 sq. units.

We say that 16 is the **square*** of 4.

i.e. $16 = 4^2$

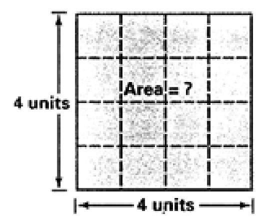


Fig. 1

In Fig. 2, the area of the square is 25 square units. What is the length of each side? The lengths of the sides of a square are equal, so if we let the length be a units, then

area of the square = $(a \times a)$ sq. units = a^2 sq. units = 25 sq. units.

But $5 \times 5 = 5^2 = 25$, so $a = 5$, i.e. the length of each side of the square is 5 units.

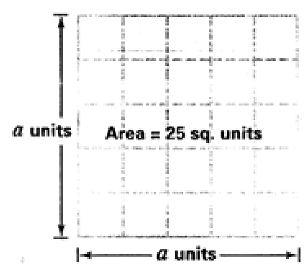


Fig. 2

TN 1 We say that 5 is a **square root*** of 25. The symbol for square root is $\sqrt[2]{\ }$, where $\sqrt{\ }$ is called the radical sign*.

Hence we write $\sqrt[2]{25} = 5$.

But very often we simply write $\sqrt{25} = 5$.

For our examples above, $\sqrt[2]{16} = \sqrt{16} = \sqrt{4^2} = 4$,

$\sqrt[2]{25} = \sqrt{25} = \sqrt{5^2} = 5$.

In general, for a positive number a, $\sqrt{a^2} = a$.

TN 2

> **DEFINITION**
> **If $a \times a = n$, then a is a square root of n.**

e.g. $2 \times 2 = 4$, therefore 2 is a square root of 4.

Think TN 3

Is it true that $\sqrt{(-4)^2} = -4$?

Section Objective

- To understand the physical meaning of the square of a number.
- To learn the meaning of the square root of a number.
- To use the radical sign to express the positive and the negative square roots of a number.

TG Teaching Guide

The meaning of the square of a number is illustrated by finding the area of a square with the length of each side given.

Conversely, when given the area of a square, students are asked to find the length of each side. Then the term 'square root' and the radical sign are introduced. After that the definition of the 'square root' of a number follows.

Teaching Notes

TN 1 Students have learned squares and square roots in primary school, but the discussion was confined to whole numbers. This section starts with whole numbers as a revision. But other sections of this chapter will cover square roots of decimals and fractions.

TN 2 Algebraically, we define the square of a number as the product of a number and itself, and we define a square root in terms of th

However, $(-2) \times (-2) = 4$ also. Therefore -2 is also a square root of 4.

Hence 2 and -2 are two square roots of 4.

4 Every *positive number* has two square roots. If the *positive number* is n, then the *positive square root* is written as $\sqrt{n}$; the *negative square root* is written as $-\sqrt{n}$.

e.g. (i) The two square roots of 4 are
$\sqrt{4} = 2$ and $-\sqrt{4} = -2$.

(ii) The two square roots of 9 are
$\sqrt{9} = 3$ and $-\sqrt{9} = -3$.

Note:
n must be positive, because at this level, we would not consider square roots (positive or negative) of any negative number, e.g. $\sqrt{-4}$ and $-\sqrt{-9}$. They are meaningless to us at this level.

Note that $\sqrt{4} = 2$ only, and $\sqrt{4} \neq \pm 2$

5 ***Note:*** We should be very careful that $\sqrt{n}$ denotes only the positive square root of n.

Historical Note

Although our symbol for 'the positive square root of 9' is $\sqrt{9}$, it has not always been written that way. Some of the symbols that have been used to mean the same thing are shown below. The last of these was used by Sir Isaac Newton, who is famous not only as a scientist, but also as a distinguished mathematician.

℞9 ℓ9 z/9 √9 $9^{½}$

CLASS PRACTICE

6 **1.** Complete the following table of the squares of the first 20 positive integers.

a	1	2	3	4	5	6	7	8	9	10
a^2	1	4	9	16	25	36	49	64	81	100

a	11	12	13	14	15	16	17	18	19	20
a^2	121	144	169	196	225	256	289	324	361	400

Note: The numbers 1, 4, 9, 16, ⋯ (i.e. $1^2, 2^2, 3^2, 4^2, \cdots$), which are the squares of natural numbers, are called **square numbers***.

2. Referring to the table in **Question 1**, find the value of each of the following:

(a) $\sqrt{36}$ = 6 **(b)** $\sqrt{49}$ = 7

(c) $\sqrt{100}$ = 10 **(d)** $\sqrt{196}$ = 14

(e) $\sqrt{289}$ = 17 **(f)** $\sqrt{400}$ = 20

(g) $\sqrt{169}$ = 13 **(h)** $\sqrt{361}$ = 19

(i) $-\sqrt{121}$ = -11 **(j)** $-\sqrt{16}$ = -4

$-\sqrt{121}$ and $-\sqrt{16}$ represent the

5.2 Evaluating Square Roots

A. Evaluating Simple Square Roots

Notice that $\sqrt{4\times 9} = \sqrt{36} = 6$
and $\sqrt{4}\times\sqrt{9} = 2\times 3 = 6,$
therefore $\sqrt{4\times 9} = \sqrt{4}\times\sqrt{9}.$

Likewise $\sqrt{4\times 25} = \sqrt{100} = 10$
and $\sqrt{4}\times\sqrt{25} = 2\times 5 = 10,$
therefore $\sqrt{4\times 25} = \sqrt{4}\times\sqrt{25}.$

These examples suggest the following:

TN 7

For positive numbers a and b, $\sqrt{a\times b} = \sqrt{a}\times\sqrt{b}$.

We may use this result to evaluate square roots.

Example 1 (a) Express 484 in prime factors using index notation.
(b) Hence evaluate $\sqrt{484}$.

Recall: *A prime number is a natural number (other than 1) that is not divisible by any natural number except 1 and itself. Prime factors of a number m are prime numbers that can divide m exactly.*

Solution

(a) $484 = 2\times 2\times 11\times 11$
$= \underline{\underline{2^2\times 11^2}}$

2	484
2	242
11	121
	11

(b) By (a), $\sqrt{484} = \sqrt{2^2\times 11^2}$
$= \sqrt{2^2}\times\sqrt{11^2}$
$= 2\times 11$
$= \underline{\underline{22}}$

$\sqrt{a\times b} = \sqrt{a}\times\sqrt{b}$ where $a = 2^2, b = 11^2$.

Alternatively,

$\sqrt{484} = \sqrt{2^2\times 11^2}$
$= \sqrt{2\times 2\times 11\times 11}$
$= \sqrt{(2\times 11)\times(2\times 11)}$
$= \sqrt{(2\times 11)^2}$
$= 2\times 11$
$= \underline{\underline{22}}$

SECTION OBJECTIVE

- To apply the rule $\sqrt{ab} = \sqrt{a}\times\sqrt{b}$ to find the square root of a composite number.
- To apply the rule $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ to find the square root of a fraction.

TEACHING GUIDE

By comparing the results of $\sqrt{4\times 9}$ and $\sqrt{4}\times\sqrt{9}$, the rule $\sqrt{ab} = \sqrt{a}\times\sqrt{b}$ is suggested.

Students should first recall how to express a composite number in prime factors. Then the technique to apply the rule $\sqrt{ab} = \sqrt{a}\times\sqrt{b}$ to evaluate the square root of a number is exemplified.

TEACHING NOTES

TN 7 Students have actually used the idea such as $\sqrt{4\times 9} = \sqrt{4}\times\sqrt{9}$ in primary school, but here we generalize it using an algebraic formula $\sqrt{a\times b} = \sqrt{a}\times\sqrt{b}$.

Here we specify that the formula only holds for positive numbers. In fact, the formula is not true when a and b are negative numbers.

EXTRA EXAMPLES

Example I

(a) [illegible]ss 44 100 in prime factors [illegible] index notation.

(b) Hence evaluate $\sqrt{44\ 100}$.

Solution

(a) 44 100
$= 2\times 2\times 3\times 3\times 5\times 5\times 7\times 7$
$= \underline{\underline{2^2\times 3^2\times 5^2\times 7^2}}$

(b) $\sqrt{44\ 100}$
[illegible]

Example 2 Evaluate $\sqrt{784}$.

Solution

$$784 = 2 \times 2 \times 2 \times 2 \times 7 \times 7$$
$$= 2^2 \times 2^2 \times 7^2$$
$$\therefore \sqrt{784} = \sqrt{2^2 \times 2^2 \times 7^2}$$
$$= \sqrt{2^2} \times \sqrt{2^2} \times \sqrt{7^2}$$
$$= 2 \times 2 \times 7$$
$$= \underline{\underline{28}}$$

◄
$$\begin{array}{r|l} 2 & 784 \\ \hline 2 & 392 \\ \hline 2 & 196 \\ \hline 2 & 98 \\ \hline 7 & 49 \\ \hline & 7 \end{array}$$

◄ *Note that*
$$\sqrt{a \times b \times c}$$
$$= \sqrt{a} \times \sqrt{b \times c}$$
$$= \sqrt{a} \times \sqrt{b} \times \sqrt{c}$$

Example 3 Evaluate $\sqrt{0.04}$.

Solution

$$\sqrt{0.04} = \sqrt{4 \times 0.01}$$
$$= \sqrt{4} \times \sqrt{0.01}$$
$$= 2 \times 0.1$$
$$= \underline{\underline{0.2}}$$

◄ $0.01 = 0.1 \times 0.1 = 0.1^2$

$\therefore \sqrt{0.01} = 0.1$

CLASS PRACTICE

Evaluate the following square roots.
(*The first one is done for you as an example.*)

1. $\sqrt{4 \times 16} = \sqrt{4} \times \sqrt{16} = 2 \times 4 = 8$
2. $\sqrt{16 \times 25} = \sqrt{16} \times \sqrt{25} = 4 \times 5 = 20$
3. $\sqrt{9 \times 49} = \sqrt{9} \times \sqrt{49} = 3 \times 7 = 21$
4. $\sqrt{11^2 \times 3^2} = \sqrt{11^2} \times \sqrt{3^2} = 11 \times 3 = 33$
5. $\sqrt{8^2 \times 10^2} = \sqrt{8^2} \times \sqrt{10^2} = 8 \times 10 = 80$
6. $\sqrt{0.3^2 \times 0.09} = \sqrt{0.3^2} \times \sqrt{0.09} = 0.3 \times 0.3 = 0.09$
7. $\sqrt{2^2 \times 3^2 \times 5^2} = \sqrt{2^2} \times \sqrt{3^2} \times \sqrt{5^2} = 2 \times 3 \times 5 = 30$

EXERCISE 5A

(Level 1)

Find the value of each of the following square roots. **[Nos. 1–8]**

① **1.** $\sqrt{256}$ 16 ① **2.** $\sqrt{441}$ 21 ① **3.** $\sqrt{900}$ 30

① **4.** $-\sqrt{324}$ −18 ① **5.** $-\sqrt{576}$ −24 ② **6.** $\sqrt{0.09}$ 0.3

② **7.** $\sqrt{0.36}$ 0.6 ② **8.** $-\sqrt{0.81}$ −0.9

Evaluate the following expressions. **[Nos. 9–11]**

③ **9.** $\sqrt{6^2 \times 8^2}$ 48 ③ **10.** $\sqrt{8^2 \times 11^2}$ 88 ③ **11.** $\sqrt{25 \times 64}$ 40

④ **12.** The area of a square is 64 cm^2. Find the length of each side of the square. 8 cm

(Level 2)

Find the value of each of the following square roots. **[Nos. 13–17]**

① **13.** $\sqrt{1\,024}$ 32 ① **14.** $-\sqrt{1\,296}$ −36 ② **15.** $\sqrt{30.25}$ 5.5

③ **16.** $\sqrt{25 + 144}$ 13 ③ **17.** $\sqrt{625 - 49}$ 24

Evaluate the following expressions. **[Nos. 18–24]**

③ **18.** $\sqrt{3^2 \times 7^2 \times 11^2}$ 231 ③ **19.** $\sqrt{2^2 \times 2^2 \times 6^2 \times 6^2 \times 10^2}$ 1 440

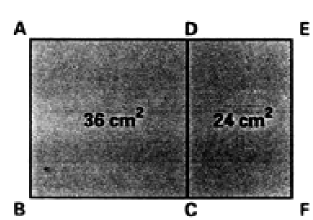
⑤ **20.** $-3 \times \sqrt{169}$ −39 ⑤ **21.** $\sqrt{81} + \sqrt{36}$ 15

⑤ **22.** $\sqrt{361} + 2 \times \sqrt{256}$ 51 ⑤ **23.** $5 \times \sqrt{0.04}$ 1

⑤ **24.** $\sqrt{1.44} - 2 \times \sqrt{0.25}$ 0.2

④ **25.** The area of a square picture is 14.44 cm^2. Find the perimeter of the picture. 15.2 cm

④ ***26.** In the figure, ABCD is a square of area 36 cm^2 and CDEF is a rectangle of area 24 cm^2. They are put side by side to form a rectangle ABFE as shown. Find the lengths of the sides AB and BF. AB = 6 cm , BF = 10 cm



ASSIGNMENT GUIDE ..... Ex. 5A

TYPE	OBJECTIVE
①	Evaluate a square root of an integer.
②	Evaluate a square root of a decimal.
③	Evaluate a square root of an expression.
④	Word problems on square roots.
⑤	Evaluate expressions involving square roots.

TYPE	Level 1 Question	Level 2 Question
①	1 – 5	13, 14
②	6 – 8	15
③	9 – 11	16 – 19
④	12	25, 26
⑤		20 – 24

SOLUTIONS

(Ex. 5A)

1. $256 = 2\times2\times2\times2\times2\times2\times2\times2$
$= 2^2\times2^2\times2^2\times2^2$
$\therefore \sqrt{256} = \sqrt{2^2\times2^2\times2^2\times2^2}$
$= \sqrt{2^2}\times\sqrt{2^2}\times\sqrt{2^2}\times\sqrt{2^2}$
$= 2\times2\times2\times2$
$= \underline{\underline{16}}$

2. $441 = 3\times3\times7\times7$
$= 3^2\times7^2$
$\therefore \sqrt{441} = \sqrt{3^2\times7^2}$
$= \sqrt{3^2}\times\sqrt{7^2}$
$= 3\times7$
$= \underline{\underline{21}}$

3. $900 = 2\times2\times3\times3\times5\times5$
$= 2^2\times3^2\times5^2$
$\therefore \sqrt{900} = \sqrt{2^2\times3^2\times5^2}$
$= \sqrt{2^2}\times\sqrt{3^2}\times\sqrt{5^2}$
$= 2\times3\times5$
$= \underline{\underline{30}}$

4. $324 = 2\times2\times3\times3\times3\times3$
$= 2^2\times3^2\times3^2$
$\therefore -\sqrt{324} = -\sqrt{2^2\times3^2\times3^2}$
$= -(\sqrt{2^2}\times\sqrt{3^2}\times\sqrt{3^2})$
$= -(2\times3\times3)$
$= -18$